Proceedings of DETC'03
ASME 2003 Design Engineering Technical Conferences and Computers and Information in Engineering Conference
Chicago, Illinois USA, September 2-6, 2003

DETC2003/DAC-48764

MULTIDIMENSIONAL SOLUTION CLUSTERING AND ITS APPLICATION TO THE COOLANT PASSAGE OPTIMIZATION OF A TURBINE BLADE

Min Joong Jeong*
Institute of Environmental Studies
Graduate School of Frontier Sciences
The University of Tokyo
7-3-1 Hongo, Bunkyo-ku,
Tokyo 113-8656, JAPAN
jeong@garlic.q.t.u-tokyo.ac.jp

Brian H. Dennis
Institute of Environmental Studies
Graduate School of Frontier Sciences
The University of Tokyo
7-3-1 Hongo, Bunkyo-ku,
Tokyo 113-8656, JAPAN
dennis@garlic.q.t.u-tokyo.ac.jp

Shinobu Yoshimura
Institute of Environmental Studies
Graduate School of Frontier Sciences
The University of Tokyo
7-3-1 Hongo, Bunkyo-ku,
Tokyo 113-8656, JAPAN
yoshi@q.t.u-tokyo.ac.jp

ABSTRACT

Data clustering methods can be a useful tool for engineering design that is based on numerical optimization. The clustering method is an effective way of producing representative designs, or clusters, from a large set of potential designs. These methods have recently been applied to the clustering of Pareto-optimal solutions from multi-objective optimization. The results presented here focus on the application of clustering to single objective optimization results. In the case of single objective optimization, the method is used to determine the clusters in a set of quasi-optimal feasible solutions generated by an optimizer. A data clustering procedure based on an evolutionary method is briefly described. The number of clusters is determined automatically and need not be known a priori. The method is demonstrated by application to the results of a turbine blade coolant passage shape optimization problem. The solutions are transformed to a lower-dimensional space for better understanding of their variance and character. Engineering information, such as the shapes and locations of the internal passages, is supported by the visualization of clustered solutions. The clustering, transformation, and visualization methods presented in this study might be applicable to the increasing interpretation demands of various optimization solutions.

*http://www.save.k.u-tokyo.ac.jp Tel:+81-3-5841-8922 Fax:+81-3-5800-6876

INTRODUCTION

Recently, various optimization methods are being used extensively in various engineering fields. Among these methods, evolution-based optimization methods such as Genetic Algorithms have many practical applications. The evolution-based methods can generate a single optimal solution, but normally produce many feasible solutions in the search for the optimum. In single objective optimization, a single optimal solution may not give enough information to help the engineer make a final design choice. That is because it is usually difficult or impossible to formulate the optimization problem to include all factors which influence the choice of a particular design, such as durability and manufacturability. Therefore, if the whole set of feasible solutions including quasi-optimal and the optimal solution are provided to engineers with proper information, the engineers can use this information to choose the best overall design. However, there is a lack of rational procedures for selecting a single feasible quasi-optimal solution out of possibly large set of feasible solutions. The procedure described here provides a method that can be used to help the engineer make effective use of all the feasible solutions provided by the optimizer.

To understand the meaning of the large amount of optimization solutions, it helps if their complexity can be reduced. Generally, two major categories of approaches are used to deal with this task [1]. In the first category, information such as the Euclidean distance between solutions is used to infer how the solutions are distributed in the multidimensional space, using various methods of clustering. The emphasis of these methods is to describe large amounts of solutions more concisely with cluster attributes or some other distributions.

The other category of approaches emphasize the reduction of dimensions, that is the reduction of the number of features necessary to describe each and all of the solutions. The idea is that often the dimensions of the original solution space are not all independent of each other, i.e. the solution data may be some complicated functions of just a few independent inherent dimensions. So, the objective is to use this reduced-dimension space to describe the solutions. Some methods belong to this category are linear principal component analysis (PCA) through such as the Karhunen-Loève (K-L) transformation [2], and neural-net implementations of PCA. These methods generally try to map the high-dimensional space to one of lower dimension.

In this study, clustering and dimensional reduction of internal passage optimization solutions of a turbine blade is performed. To carry out this task, the clustering analysis in a 90-dimensional design parameter space provides the structure of the solutions. Moreover the K-L transformation is used to project the 90-dimensional solutions into a more lower-dimensional space for easy understanding of the variance and characteristics of the solutions. Engineering information of the shapes of the internal passages is supported by the visualization of clustered solutions. The clustering, transformation, and visualization methods presented in this study might be applicable to the increasing interpretation demands of various optimization solutions.

DESIGN OPTIMIZATION OF COOLANT PASSAGES OF A TURBINE BLADE

With a perpetual goal of increasing thermodynamic efficiency of turbines, various blade-cooling systems have been used. However, with the extremely high temperatures of the combustion gases it became apparent that film cooling causes increased production of NOx as well as a decreased in the aerodynamic efficiency of the turbine blade. As a remedy, a high-pressure closed-circuit internal cooling concept [3] became attractive again decades after its inception. Moreover, circular cross-section straight-through coolant passages became attractive because of the ease of their manufacturing thus lower cost of such blades. An intuitive approach became to place a large number of natural cooling networks appearing in biological systems. However, the problem that has not been answered yet is where precisely to locate each such coolant passage and what should be the radius of each individual passage.

Turbine Design Objective and Constraints

The design objective is to minimize the total amount of heat transferred to the blade (integrated heat flux on the hot surface of the blade) while maintaining a maximum temperature, T_{max}, which is lower than the maximum allowable temperature, T_{allow}. This objective indirectly minimizes the amount of coolant required to cool the blade. The minimization of this objective could result in the reduction of the number of cooling passages as well.

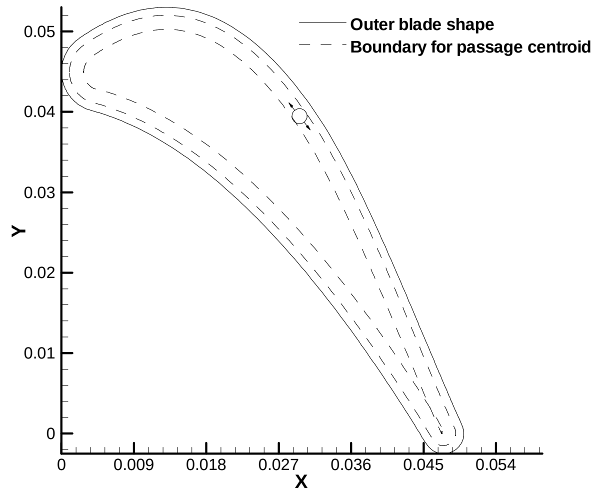

Figure 1. REGION WHERE COOLANT PASSAGE CENTERS ARE ALLOWED.

The objective function is computed by integrating heat flux across the blade outer surface, Γ. Mathematically, the objective function F is expressed as

$$F = \int_{\Gamma} k \frac{\partial T}{\partial n} dxdydz \tag{1}$$

where T is the blade temperature, n is the direction normal to the surface Γ, and the constant k is the heat conduction coefficient for the blade material. There are two inequality constraints that are expressed as

$$G_1 = \frac{T_{allow} - T_{max}}{T_{allow}} \tag{2}$$

$$G_2 = \sum_{i=1}^{nholes} C_i \tag{3}$$

where $nholes$ is the number of passages and C_i is a positive number when the distance between passage i and another passage is less than a specified distance. Otherwise the value of C_i is zero.

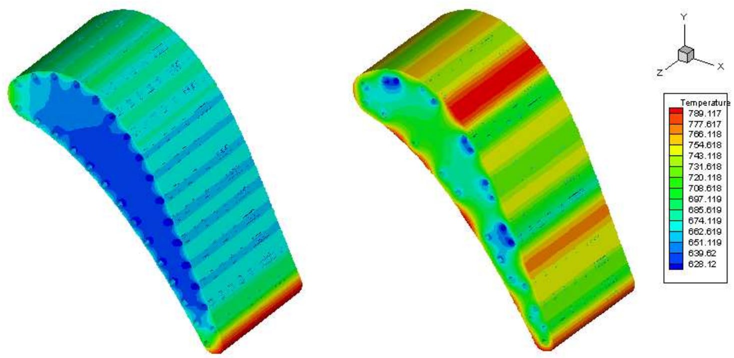

Figure 2. TEMPERATURE DISTRIBUTION FOR INITIAL DESIGN (LEFT) AND THE OPTIMIZED BEST DESIGN (RIGHT) FOR INITIALLY 30-PASSAGE CASE.

The first constraint is necessary so that the maximum temperature in the blade material is always below the maximum allowed temperature. The second constraint is needed to insure that the optimizer only searches for valid geometries. The constraints are satisfied if $G_1 \leq 0.0$ and $G_2 \leq 0.0$.

Design Parameterization

The outer blade shape is considered to be fixed and to be provided by the user at the beginning of the design optimization. Presumably, this is the blade shape that has already been optimized for its aerodynamic performance [4]. The design variables include the radius of each circular passage, r_i, and position of the passage center, $< x_i, y_i >$, in the blade cross-section. The passage center is allowed to move normal to the outer contour within a specific region as shown in Fig. 1. The design variable x_i is a distance in the direction normal to the blade surface and is non-dimensionalized so that it always lies between the two dashed lines shown in Figure 4. The variable y_i is a non-dimensional distance in a surface following coordinate direction that is taken along the outer surface of the blade. For 30 passages, this parameterization leads to a total of 90 variables. The passage radius, r_i, is set to zero if it goes below a specified value, r_{min}, thereby allowing the optimizer to reduce the total number of passages.

$$r_i = \begin{cases} 0 & r_i < r_{min} \\ r_i & r_i \geq r_{min} \end{cases} \tag{4}$$

This mixed continuous/discrete behavior creates a discontinuity in the objective function space and makes the problem difficult for classical optimization algorithms to solve. A triangular surface mesh [5] and a tetrahedral volume mesh were generated automatically for each candidate design. The mesh generator did an adequate job of placing enough points between the passages and the blade surface, even when the passages were very close to the surface. A typical mesh had around 70,000 nodes.

.

Table 1. DESIGN VARIABLE BOUNDS.

Parameter	Lower bound	Upper bound
r_i	0.25 mm	0.8 mm
x_i	1.0 mm	2.75 mm
y_i	$\frac{(i-1)}{nholes}$	$\frac{i}{nholes}$

Table 2. CONSTRAINTS USED FOR THE PASSAGES DESIGN.

Maximum allowable temperature, T_{allow}	800.0°C
Blade heat conduction coefficient, k	7.0 $W/m-°C$
Minimum passage radius, r_{min}	0.5 mm
Blade axial chord length	5.0 cm
Minimum allowable distance between passages	0.1 mm

Design Optimization

In this section, design optimization of the coolant passages using the IOSO optimization method [6, 7] is presented. The design variable bounds were set according to Table 1. Additional constants are shown in Table 2. The outer blade geometry was created by generating a series of 2-D turbine airfoils [8] and stacking the sections along the blade spanwise direction. Thermally insulated conditions were used on the blade end surfaces. Convective heat transfer (Robin type) boundary conditions were used on the surfaces of the coolant passages and on the outer blade surface.

The maximum number of coolant passages was set to 30. The total number of design variables was 90. This optimization problem was solved using IOSO algorithm. The IOSO ran on 54 processors in a PC cluster composed of Pentium II and Pentium III processors. 40 simultaneous analyses were run per iteration. That is, the design population size was 40. Each finite element heat conduction analysis used 2 processors. The IOSO method requires only a single tunable parameter. That parameter controls the depth of the global search, and the parameter was set for an extensive global search. The convergence criteria for the IOSO method were met by iteration 40 and the process was terminated.

Finally, 279 feasible solutions were obtained out of the optimization iterations. The outer surface temperature on the optimized design is much closer to T_{allow} than in the initial design as shown in Fig. 2. The best possible design could be achieved if the entire outer surface temperature would be equal to T_{allow}. In that case, the smallest possible integrated heat flux would be 1038.0 Watts. Typically, only this one solution is often provided to engineers. However, any information about the other 278 solutions is normally not used by engineers due to the large size of the data. We hope that by using clustering and dimensional reduction techniques this data can be transformed into useful information for choosing a final design.

CLUSTERING ALGORITHMS

Clustering has been effectively used various engineering and scientific fields such as psychology, biology, medicine, computer vision, communications, and remote sensing. Extensive overviews of clustering algorithms can be found in literature [9–11]. The primary objective of the clustering is to classify a given data set of multidimensional vectors (solutions) into several homogeneous clusters. It seems very profitable to apply the clustering to the solutions derived from optimization. If the solutions are appropriately classified into several clusters, it is expected that engineers can interpret mathematical as well as engineering characteristics of the solutions in a more abstract manner. They may choose a final solution that best meets the overall goals of the design.

Clustering algorithms basically aim at minimizing the following clustering function :

$$E(\mathbb{X},U,V) = \sum_{k=1}^{K}\sum_{i=1}^{n} {u_{ik}}^m \cdot dis(X_i, v_k) \tag{5}$$

where $\mathbb{X} \in X_i \subseteq \mathbb{R}^d$ is a whole data set in a d-dimensional real-valued space, n is the number of all data patterns to be clustered, K is the number of clusters, $u_{ik} \in [0,1]$ is the membership degree of x_i belonging to the k-th cluster, and v_k is the center of the k-th cluster, i.e. the search vector. $dis(X_i, v_k)$ means the Euclidean distance between X_i and v_k. The parameter $m > 1$ is called the fuzziness index. For $m \rightarrow 1$, the clusters tend to be crisp, i.e. either $u_{ik} \rightarrow 1$ or 0. The function has two constraints those are (a) it should be guaranteed that none of the clusters is empty, and (b) it should be ensured that for each datum, the sum of its membership degrees to all clusters has to be 1. We are required to find the optimum membership degrees u_{ik} and the optimum cluster search vectors v_k. The membership degree u_{ik} and the cluster search vector v_k are defined as follows:

$$v_k = \frac{\sum_{i=1}^{n} {u_{ik}}^m \cdot X_i}{\sum_{i=1}^{n} {u_{ik}}^m} \tag{6}$$

and

$$u_{ik} = \left[\sum_{j=1}^{K} \left(\frac{\|X_i - v_k\|^2}{\|X_i - v_j\|^2} \right)^{\frac{1}{1-m}} \right]^{-1} \tag{7}$$

As summary, the clustering process means that for a given data set $\mathbb{X}$, we search u_{ik} and v_k that minimize E.

Evolutionary Clustering Algorithm

There are many iterative algorithms to minimize E, such as the K-Means Algorithm (KMA) [9] for example. In every iteration of KMA, according to Eq. (7), each data pattern is assigned the closest search vector. Then, the search vectors are calculated using Eq. (6), i.e. as the mean vectors of the assigned patterns belonging to the corresponding clusters, respectively. The iteration terminates when the following criteria are satisfied, i.e. (a) there is no reassignment of any pattern from one cluster to another or (b) the E value ceases to decrease. However, one serious problem of the KMA algorithm is that the clustering result depends on the initial search vectors [12, 13]. When improper initial search vectors are chosen, the calculation can become trapped in local minimum, and an ill-clustered result is obtained.

To bypass the local minima issue, an Evolutionary Clustering Algorithm (ECA) was used for clustering. Its principal idea is as follows: At first a collection, i.e. population, of possible solutions encoded as parameter vectors are prepared. These correspond to chromosome in the evolutionary strategy [14, 15]. From this population, a new population, i.e. the next generation, is generated through the following genetic process. Offspring are produced from the old chromosomes. The offspring are produced by mixing genes of different chromosomes (an intermediate tendency recombination) and sometimes by changing some genes of the chromosomes randomly (mutation). Then the best chromosomes are deterministically selected for the next generation. The details of the ECA is found in Refs [16]. The ECA typically shows better clustering results than the KMA and the KMA-related algorithms and takes less computational effort than other evolution-based algorithms.

The Davies-Bouldin Index

Although the ECA provides good clustering results, there is one more problem to apply clustering to interpret the optimization solutions. That is how we decide the number of clusters. Since the solutions are normally in a multidimensional parameter or function space, any direct selection of the cluster number is seldom known a priori. Therefore we adopted a mathematical approach to help determine the number of clusters, mainly the Davies-Bouldin (DB) index [17]. The DB index was originally proposed as a way of deciding when to stop clustering solutions.

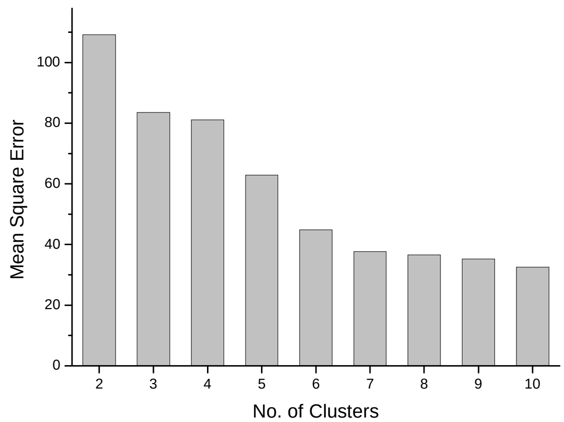

Figure 3. CLUSTERING FUNCTION VALUE VERSUS CLUSTER NUMBER.

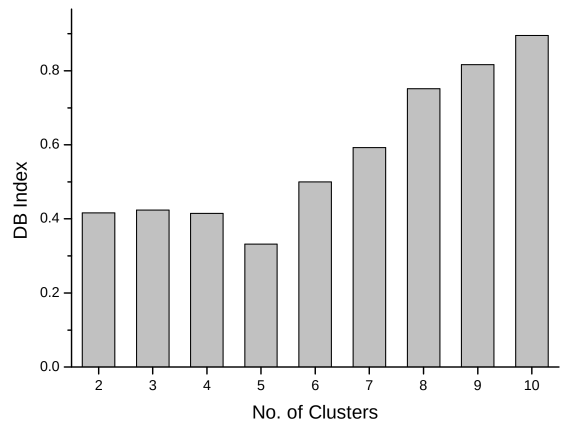

Figure 4. DB INDEX VERSUS CLUSTER NUMBER.

The index is plotted against the number of clusters and clustering is stopped when the index is minimized. The index does not depend on either the number of clusters or the clustering method. This is advantageous to apply to the clustering of the optimization solutions. Given a partition of solutions into K clusters, one first defines the following measure of within-to-between cluster spread for all pairs of clusters (j,k).

$$R_{j,k} = \frac{e_j + e_k}{m_{j,k}} \tag{8}$$

where e_j is the average error for the jth cluster and $m_{j,k}$ is the Euclidean distance between the centers of the jth and kth clusters. The index for the kth cluster is

$$R_k = \max_{j \neq k} \{R_{j,k}\} \tag{9}$$

and the Davies-Bouldin index for the K-cluster clustering is

$$DB(K) = (1/K) \sum_{k=1}^{K} R_k \tag{10}$$

The index is supposed to show significant low level when it is applied to a proper cluster number. Therefore we can find the cluster number using the comparison of each cluster number and its DB index.

CLUSTERING THE COOLANT SHAPE SOLUTIONS AND THEIR VISUALIZATION

In this section, we show the procedure of interpretation of the complex passage solutions. The clustering, transformation, and visualization are used to interpret the 279 90-dimensional solutions.

At first, we found proper clusters using the ECA. Figure 3 shows each clustering value by cluster number. To give the cluster number, we calculated the DB index in 10 clusters. The DB index values on the cluster number is shown in Fig. 4. The lowest DB index value is indicated at the cluster number 5. Therefore, we fixed the cluster number as five.

Although the ECA and DB index shows that there are five clusters in the 90-dimensional space quantitatively, it is still hard to understand the distribution and structure of the solutions. Visualization of solutions permits assessment of the variance in solutions and understanding the geometrical characteristic of solutions. However, such the large 90-dimensional space makes the visualization difficult. Therefore, to check the clustered solutions and the distributed character, we transformed the solution in the 90-dimensional parameter space to a lower-dimensional space. The Karhunen-Loève transformation used for this purpose.

The Karhunen-Loève (K-L) transformation

The K-L transformation is concerned with explaining the data structure through a few linear combinations of variables. The linear combinations represent the selection of a new coordinate system and the visualization of the solutions in a reduced dimension. Let us consider the coolant passage solutions. We have the solutions those can be represented by $\mathbb{X} = \{X_1, X_2, \cdots, X_{279}\}$. Where X_i is a ith 90-dimensional solution, given by $X_i = (x_{i1}, x_{i2}, \cdots, x_{i90})^T$.

To find the principal components in a new coordinate sys-

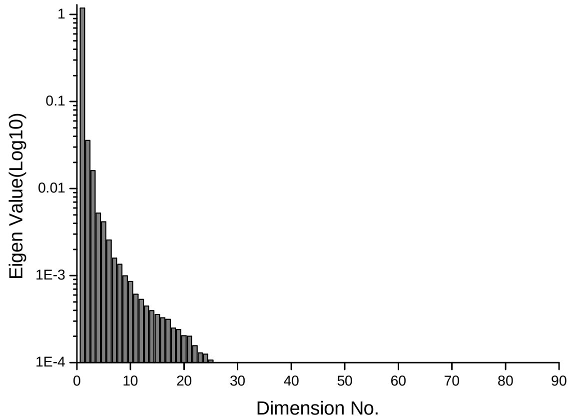

Figure 5. EIGENVALUE ON DIMENSION NUMBER.

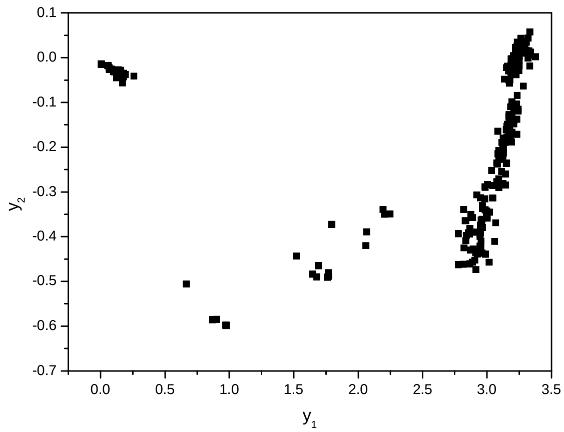

Figure 6. TWO-DIMENSIONAL REPRESENTATION OF 90-DIMENSIONAL SOLUTIONS.

tem, we need to obtain the eigenvectors Φ_i and the eigenvalues λ_i of the covariance matrix Σ_X of $\mathbb{X}$.

$$\Sigma_X \Phi_i = \lambda_i \Phi_i \tag{11}$$

With this accomplished, the 90-dimensional solutions $\mathbb{X}$ are expressed in terms of their principal components as:

$$\mathbb{Y} = \Phi^T \mathbb{X} \quad \text{with components} \quad Y_i = \Phi_i^T X_i \tag{12}$$

From the principal components, we can plot the solutions in the lower-dimensional space and get a graphical feel of their distribution. Figure 5 shows the eigenvalues of the turbine solutions. As shown in Fig. 5, the first and second components have 97%

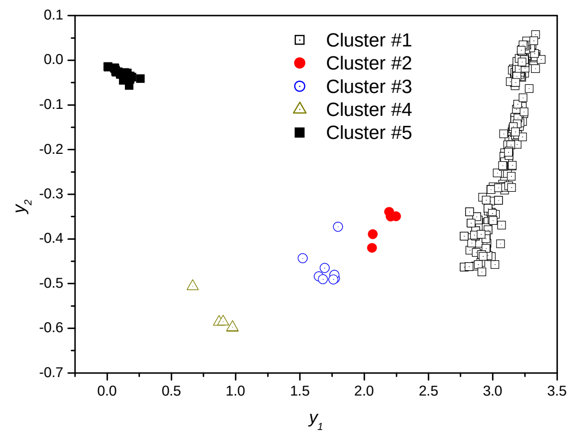

Figure 7. FIVE CLUSTERS IN THE TWO-DIMENSIONAL SPACE.

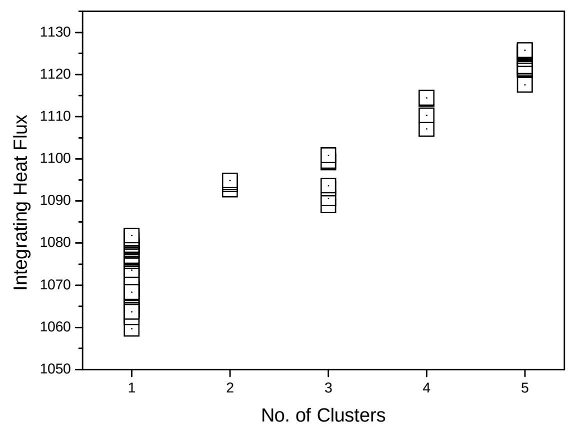

Figure 8. OPTIMIZATION FUNCTION VALUE ON EACH CLUSTER NUMBER.

of the whole eigenvalues. Therefore it is clear that only two new dimensions of the transformed solutions are enough to represent the main variation and characteristics of the solutions. According to Eq.12, the two dimensions are given by

$$\begin{aligned} y_1 = {} & 0.2775 \cdot x_{66} + 0.259 \cdot x_{39} + 0.244 \cdot x_6 + \\ & \cdots + 6.362 * 10^{-6} \cdot x_{61} + 3.737 * 10^{-6} \cdot x_{34} \end{aligned} \tag{13}$$

$$\begin{aligned} y_2 = {} & -0.429 \cdot x_{75} + 0.363 \cdot x_{18} - 0.324 \cdot x_{84} + \\ & \cdots - 1.670 * 10^{-5} \cdot x_{37} - 6.233 * 10^{-6} \cdot x_{88} \end{aligned} \tag{14}$$

The above two dimensions are the linear combinations of the original dimensions. It is ascertainable from the new dimensions

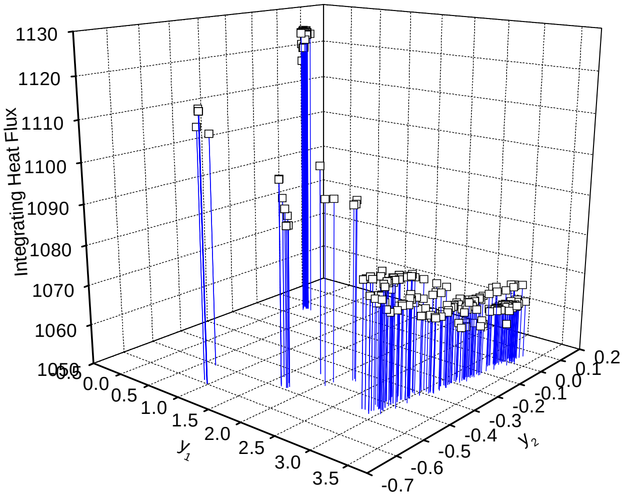

Figure 9. OPTIMIZATION FUNCTION VALUE WITH TWO-DIMENSIONAL PARAMETER VALUE.

that what kinds of original dimensions are important to the optimization problem. For example, in Eq. 13 and 14, we can identify the dimension x_{66} and x_{75} have much influence on the optimization while the x_{34} and x_{88} have less one.

The 2-dimensional representation is shown in Fig. 6. In Fig. 6, the solutions are dispersed but have several clusters. When we represent the clustering result through the 2-dimensional principal component space in Fig. 7, there are five distinct clusters. These clusters are the same as those of the five clusters in the 90-dimensional space. Therefore we can say that the clustering result is also qualitatively suitable. The first cluster has 207 solutions which are 74.2% of the solutions. The second, third, forth, and fifth have 5, 16, 14, and 37 solutions respectably. As shown in Fig. 7, the five clusters are well separated with other clusters, and the solutions of each clusters are tightly gathered.

To examine the relations between clusters and the function values for the integrated heat flux, we plotted the function values vertically on Fig. 8. The function values are decreased by stratum. Figure 9 shows the 2 principal components of parameters and the function values. As shown in Fig. 9, the cluster #1 has the optimal solution and many quasi-optimal solutions. Although the cluster #5 has feasible solutions, it consists of the highest function values. The function values of each cluster are decreased from the cluster #5 to #1. The discrete relations between clusters are considered as the product of the iteration process of optimizer. However, the information of the relations between the clusters and their function values will support engineers to understand the relationship between the function and parameter space. The information is irrelevant to their number of solutions or dimensions.

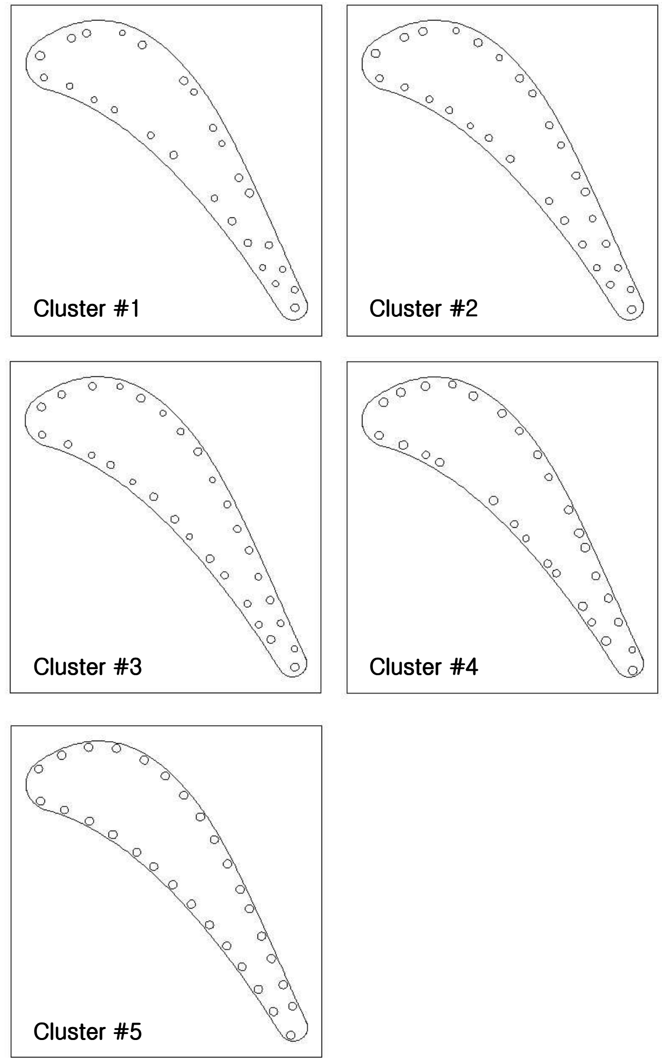

Figure 10. COOLING PASSAGES OF EACH CLUSTER.

Visualization of cooling passage shapes

Based on the clustering results, the actual passage shapes of cluster centers are shown in Fig. 10. The shape of the cluster #1 has 26 passages which are the fewest of those of five clusters. The cluster #3 and #5 have 30 passages, which is the same number as the initial design. The cluster #2 and #4 have 29 passages. The cluster #1 has the lowest objective function values, and the shape of passages is very different from the initial design. On the other hand, cluster #5 has the highest function values and isn't well developed in its passage shapes. It seems likely that the solutions in cluster #5 were generated early in IOSO optimization process whereas the solutions in #1 were probably generated during the final iterations. We can see that the clusters show how the character of the solutions are developed during the optimization process.

Although the passage shape from cluster #3 has the very similar shape and even the same passage number as the cluster #5, its function values are 34.1% lower than those of the cluster #5 in average. This shows the possibility of decreasing the objective without dramatic shape change such as in cluster #1. This is an example of the information that can be gleaned from the representative designs generated by the proposed methodology. When we provide only one solution (for example, the best function solution) to engineers, they won't have a better sense of the relationship between shape and objective function value that this information can provide, like Fig. 10. Providing a set of solutions is potentially useful if, for example, the optimal design geometry is not good for manufacturing or durability reasons.

We may think engineers are mainly concerned about the objective function value, so they will likely prefer the solutions in cluster #1. Although the solutions of the cluster #1 are concentrated in a small design parameter space, there may be some significant shape variations. Therefore, we carried out clustering of the cluster #1. Figure 11 shows two subclusters of the cluster #1. The two subclusters don't have large variation in their function values. When we look the shapes in Fig. 11, the passages of the suction side (convex) of the blade are similar to each cluster. However the passages of the pressure side (concave) are dissimilar between two subclusters. The passage number varies from 28 in subcluster #1 to 25 in subcluster #2. The objective function of the cluster #1 is mainly affected by the passage arrangement of the upper suction side. Therefore, engineers can have more freedom with the blade passage shape on the suction side. This results shows that the clustering technique can be used to provide a basic sense of sensitivity of the objective function to the passage shape.

CONCLUSIONS

A robust interpretation procedure that utilizes the recent optimization solutions has been developed using clustering and dimension transformation algorithms. The evolutionary clustering algorithm and the K-L transformation are used to explore the structure and characteristics of solutions and to find the correlations between a parameter space, a function space, and an actual design space.

The interpretation of 90-dimensional optimization solutions of turbine blade passages has been demonstrated. The clustering analysis of a 90-dimensional-design parameter space provides the structure of five clusters in the 90-dimensional space quantitatively. The transformed 2-dimensional space qualitatively shows the features of variances, structures, and correlations between the function and parameter space of the solutions. Such information could help engineers to understand the actual design shapes extracted from the function and parameter space.

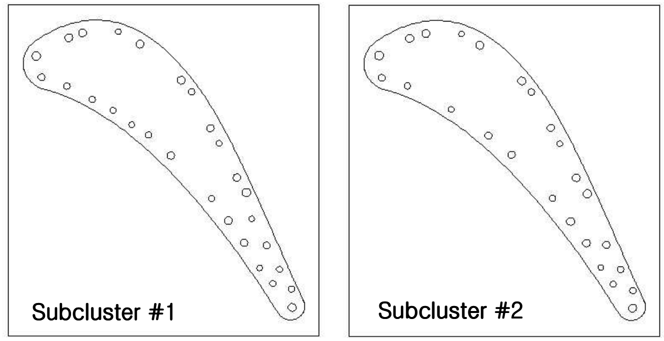

Figure 11. TWO SUBCLUSTERS OF THE CLUSTER #1.

The interpretation methods presented in this study might be applicable to the increasing interpretation demands in the general area of design optimization.

REFERENCES

[1] Meng, Z., and Pao, Y. H., 2000. "Visualization and self-organization of multidimensional data through equalized orthogonal mapping". *IEEE Transactions on Neural Networks*, **11** (4) , pp. 1031–1038.

[2] Fukunaga, K., and Koonts, W. L. G., 1970. "Application of the karhunen-loève expansion to feature selection and ordering". *IEEE Transactions on Computers*, **19** , pp. 311–318.

[3] Nomoto, H., Konga, A., Ito, S., Fukuyama, Y., Otomo, F., Shibuya, S., Sato, M., Kobayashi, Y., and Matsuzaki, H., 1997. "The advanced cooling technology for the 1500 c class gas turbine: Steam-cooled vanes and air-cooled blades". *ASME Journal of Engineering for Gas Turbines and Power*, **119** , pp. 624–632.

[4] Dennis, B. H., Egorov, I. N., Han, Z.-X., Dulikravich, G. S., and Poloni, C., 2001. "Multi-objective optimization of turbomachinery cascades for minimum loss, maximum loading, and maximum gap-to-chord ratio". *International Journal of Turbo & Jet-Engines*, **18** (3) , pp. 201–210.

[5] Shewchuk, J. R., 1996. "Triangle: Engineering a 2d quality mesh generator and delaunay triangulator". *First Workshop on Applied Computational Geometry, Philadelphia, PA* .

[6] Dennis, B. H., Egorov, I. N., Dulikravich, G. S., and Yoshimura, S., 2003. "Optimization of a large number of coolant passages located close to the surface of a turbine blade". *In: Proceedings of Turbo Expo 2003, paper GT2003-38051, 2003 ASME Turbo Expo, June 16-19, Atlanta, GA* .

[7] Egorov, I. N., 1998. "Indirect optimization method on the basis of self-organization". *Curtin University of Technology, Perth, Australia., Optimization Techniques and Appli-*

cations (ICOTA'98), **2** , pp. 683–691.

[8] Dennis, B. H., Han, Z.-X., and Dulikravich, G. S., 2001. "Optimization of turbomachinery airfoils with a genetic/ sequential quadratic programming algorithm". *AIAA Journal of Propulsion and Power,* **17** (5) , pp. 1123–1128.

[9] Jain, A. K., and Dubes, R. C., 1989. *Algorithms for Clustering Data*. Prentice-Hall, Englewood Cliffs, New Jersey.

[10] Mirkin, B., 1996. *Mathematical Classification and Clustering*. Kluwer Academic Publishers, New York.

[11] Höppner, F., Klawonn, F., Kruse, R., and Runkler, T., 1999. *Fuzzy Cluster Analysis*. John Wiley & Sons Ltd, West Sussex, England.

[12] Pedrycz, W., and Walwtzky, J., 1997. "Fuzzy clustering with partial supervision". *IEEE Transactions on Systems, Man, and Cybernetics,* **27** (5) , pp. 787–795.

[13] Baraldi, A., and Blonda, P., 1999. "A survey of fuzzy clustering algorithms for pattern recognition - part i". *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics,* **29** (6) , pp. 778–786.

[14] Bäck, T., 1996. *Evolution Algorithm in Theory and Practice*. Oxford University Press, New York.

[15] Goldberg, D. E., 1989. *Genetic Algorithms in Search Optimization and Machine Learning*. Addison-Wesley, Reading, MA.

[16] Jeong, M. J., and Yoshimura, S., 2002. "An evolutionary clustering approach to pareto solutions in multiobjective optimization". *Proceedings of 28th Design Automation Conference: ASME Design Engineering Technical Conferences, September 29, 2002 - October 2, 2002, Montreal, Canada* .

[17] Davies, D. L., and Bouldin, D. W., 1979. "A cluster separation measure". *IEEE Transactions on Pattern Analysis and Machine Intelligence PAMI,* **1** , pp. 224–227.