# MULTI-OBJECTIVE OPTIMIZATION OF TURBOMACHINERY CASCADES FOR MINIMUM LOSS, MAXIMUM LOADING, AND MAXIMUM GAP-TO-CHORD RATIO


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# Introduction

Our goal is to use an optimization method together with a CFD code design the shape of a retrofit gas turbine cascade with improved aerodynamic performance.

Given an initial cascade shape, we want to find an improved shape that will still meet the strict constraints required for the good performance of the other stages. Ideally, one should be able to remove a blade based on the old airfoil and replace it with the one designed by optimization and see an improved efficiency for the entire turbomachine.

This problem was studied in the past using a single objective GA. Here we consider a multiobjective optimization with optimization based on response surface methods.

We use a real airfoil, designed by inverse design methods, and tested experimentally at VKI. This VKI airfoil is considered to be very efficient as is.

# Problem Description

In this optimization problem, we wish to improve the efficiency of a given turbine cascade,in this case the VKI airfoil, subject to several constraints that should be satisfied to maintain the efficiency of later stages in the machine.

We assume an annular stator finite cascade with the following conditions:

inlet total temperature $T_{01}$ = 278 K

inlet total pressure $p_{01}$ = 430000 Pa

exit average static pressure $p_2$ = 101300 Pa

inlet flow angle = 1.9 degrees

span radius = 223.8 mm

number of airfoils = 45

# Objectives

| | OBJECTIVES |
|---|---|
| MAXIMIZE | Total loading force |
| MINIMIZE | Total pressure loss |
| MINIMIZE | Number of airfoils |

# Constraints

| CONSTRAINTS | Values |
|---|---|
| Total loading | > 186599 N |
| Mass flow rate (per unit span) | = 384 kg $m^{-1}$ $s^{-1}$ |
| Exit flow angle | = -70° |
| Airfoil cross-section area | = 108.8 $mm^2$ |
| Airfoil thickness distribution | > specified minimum thickness distribution |
| Airfoil trailing edge radius | > 0.5 mm |
| Fixed axial chord | |

# Objective Functions

$$F_1 = p_o^{outlet} - p_o^{inlet} + c_1\left(\frac{-70.0-\theta}{-70.0}\right)^2 + c_2\left(\frac{384.0-\dot{m}}{384.0}\right)^2$$
$$+ c_3\left(\frac{0.0001088 - A}{0.0001088}\right)^2 + c_4\left(\frac{186599 - L}{L}\right)^2 + c_5(td)^2$$

$$F_2 = -L + c_1\left(\frac{-70.0-\theta}{-70.0}\right)^2 + c_2\left(\frac{384.0-\dot{m}}{384.0}\right)^2$$
$$+ c_3\left(\frac{0.0001088 - A}{0.0001088}\right)^2 + c_4\left(\frac{186599 - L}{L}\right)^2 + c_5(td)^2$$

$$F_3 = nb$$

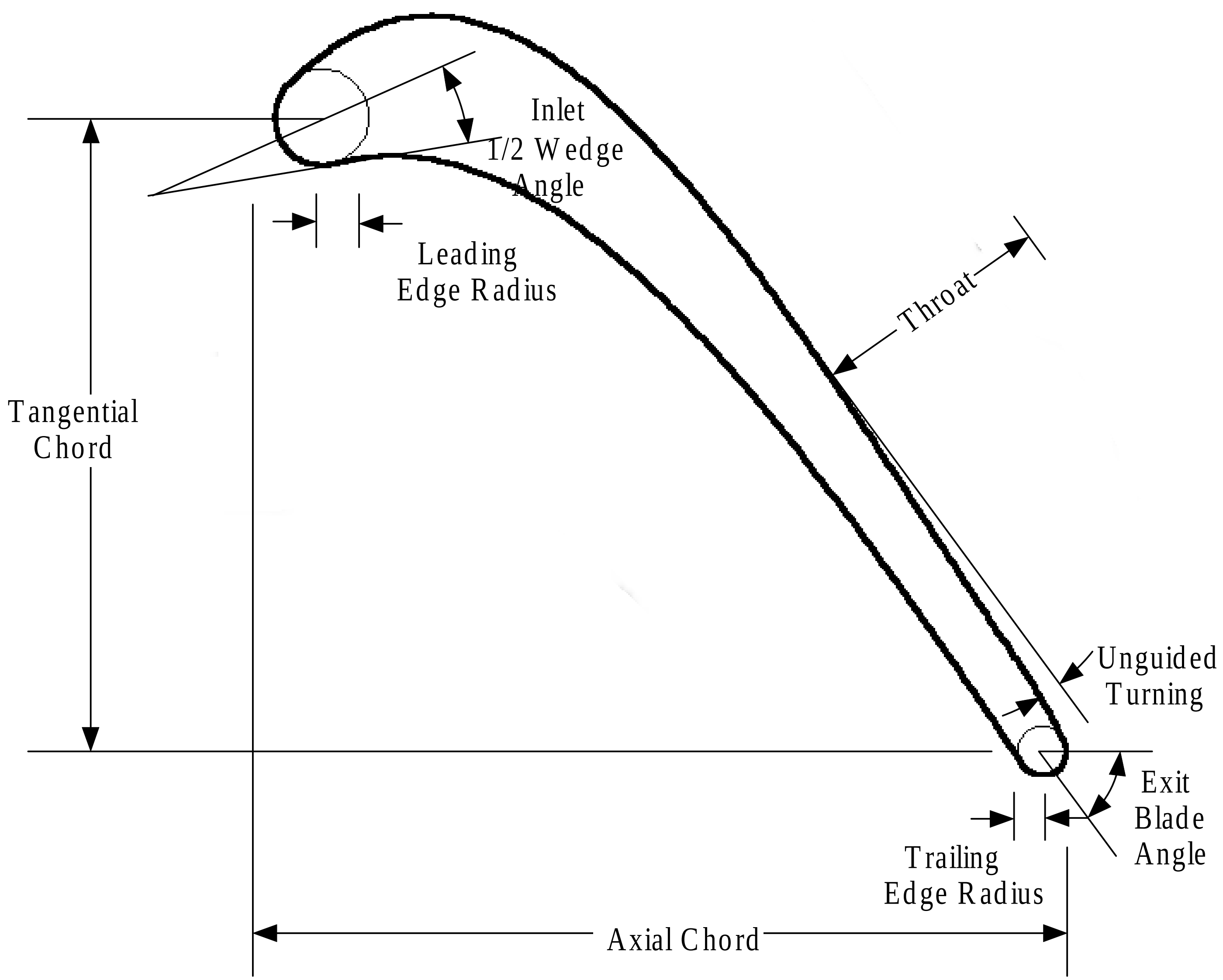
# Gas Turbine Airfoil Parameterization

- parameters are geometric quantities that are used by turbine designers
- locate five points, then use circular arcs and cubic splines to construct the airfoil shape
- small number of variables(10) needed to describe a wide range of reasonable turbine airfoil shapes

# Gas Turbine Airfoil Parameterization cont.

- initial shape generated using 10 parameters
- a b-spline was placed around the airfoil for more detailed control of the shape
- b-spline used 4 controls points on suction side and 4 control points on pressure side, each point moved normal to the surface
- total of 18 design variables

# Gas Turbine Parameterization Cont.



# Optimization Method

- Stochastic optimization algorithm combine with a response surface approach
- Capable of handling multiple objectives
- Does not require derivatives of objective function or constraints
- Can handle mixed continuous and discrete design variables

# Optimization Method Cont.

- Method is based on modified version of IOSO(Indirect method of Optimization based on Self-Organization)
- This approach allows for low number of trial points to build the initial approximation(30-50 samples for 100 variables)
- This approach also allows for improvement of the response surface in a localized search area

# Optimization Method Cont.

The optimization method basically works as follows:

1. Build initial approximation based on a given sample set
2. Use stochastic optimization method to find the minimum of the approximation to get a new design
3. Evaluate the new design with the full analysis code
4. Use the new results to improve the accuracy approximation in the local search area
5. Goto 2. until termination criteria is met

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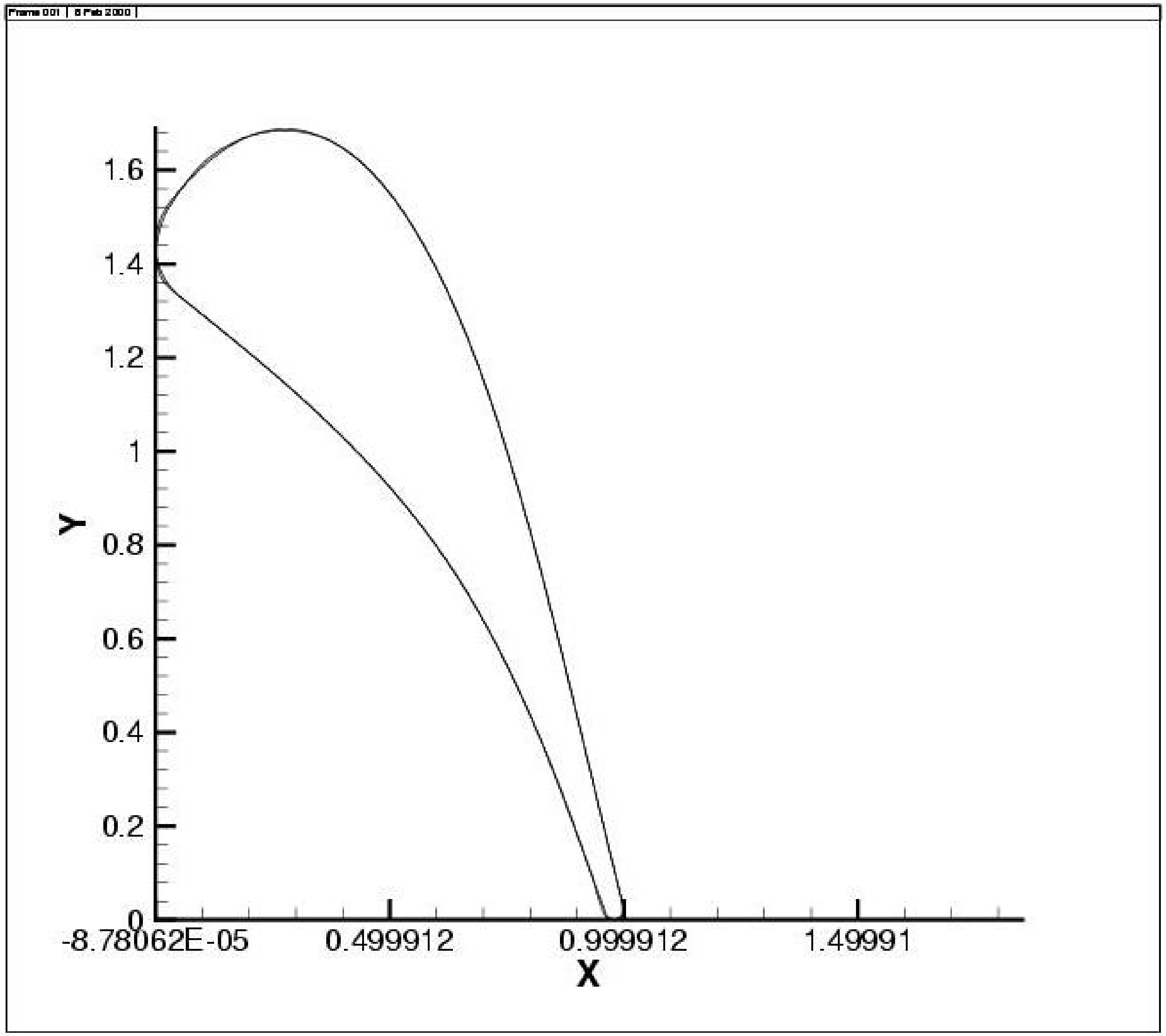
# Initial Design and Design Variable Ranges

- Initial design was found by determining the values of the design variables that best fit the original VKI airfoil data
- Design variable ranges were set to +/- 25% of the initial design variables
- Initial design was given to the optimizer at the start of the optimization

# Comparison of Original VKI Data and Parameterized VKI Airfoil



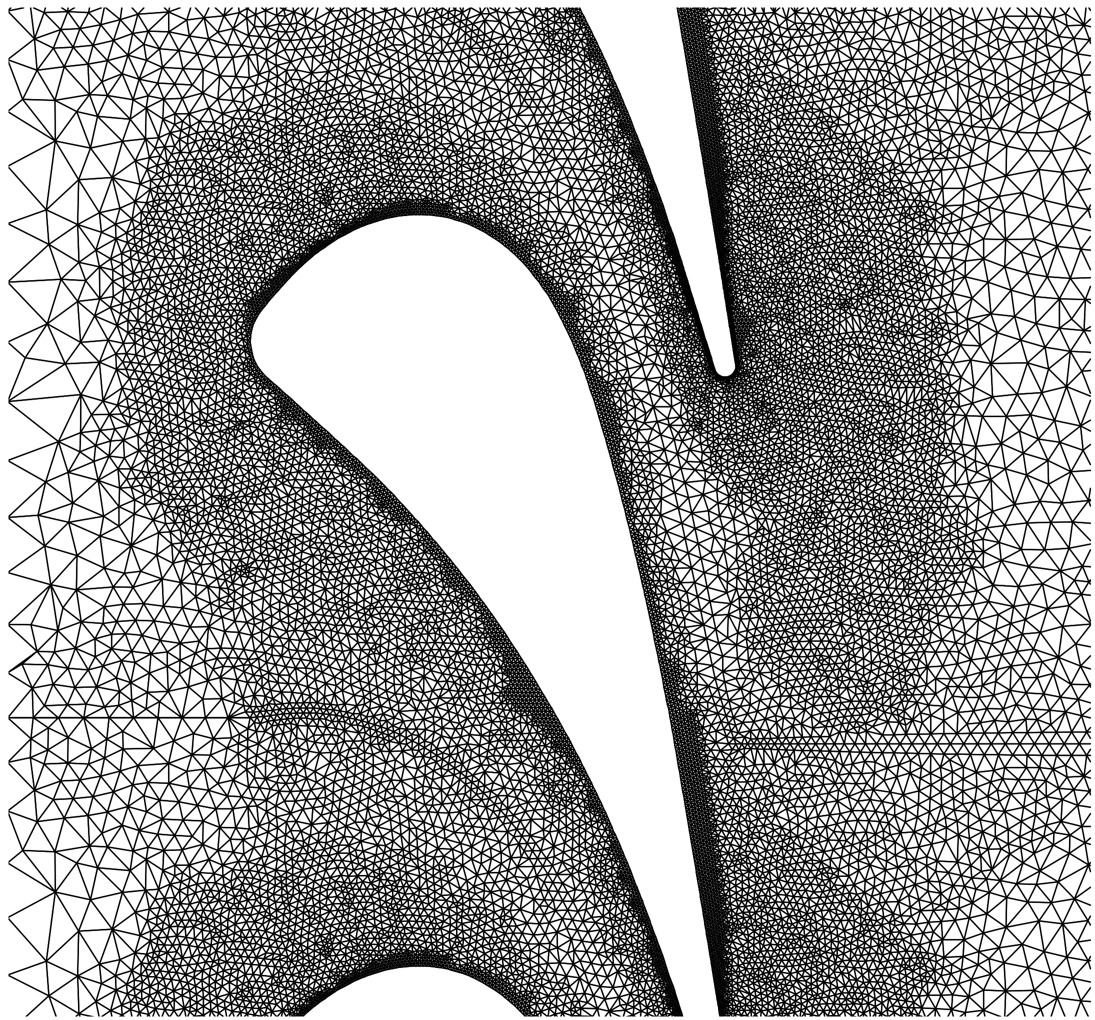
# Flow Solver

- Compressible Navier-Stokes with k-ε turbulence model
- finite volume discretization
- uses unstructured triangular grid
- Reduces the residual by five orders of magnitude in 1200 iterations with a CFL = 1.8
- Typical flow analysis takes 20 min. on Pentium II 400 MHz processor

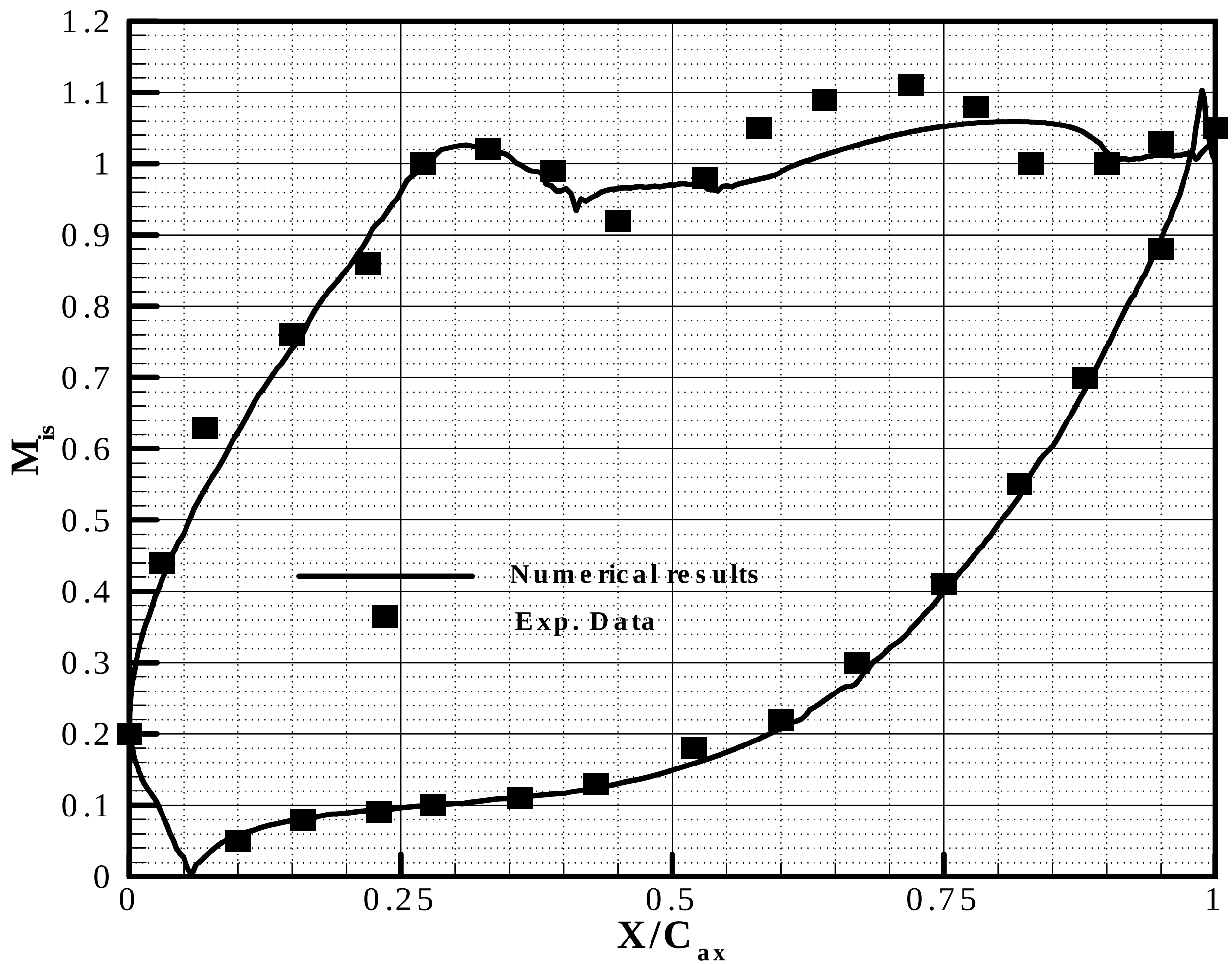
# Why unstructured?

- can handle very complex shapes
- can generate quality grids over complex objects automatically(point insertion based Delauney triangulation)
- can automatically adapt mesh to the solution when high accuracy is needed for detailed design

# Typical Unstructured Grid for Cascade Analysis



# Comparison of Computed and Measured Surface Isentropic Mach Number

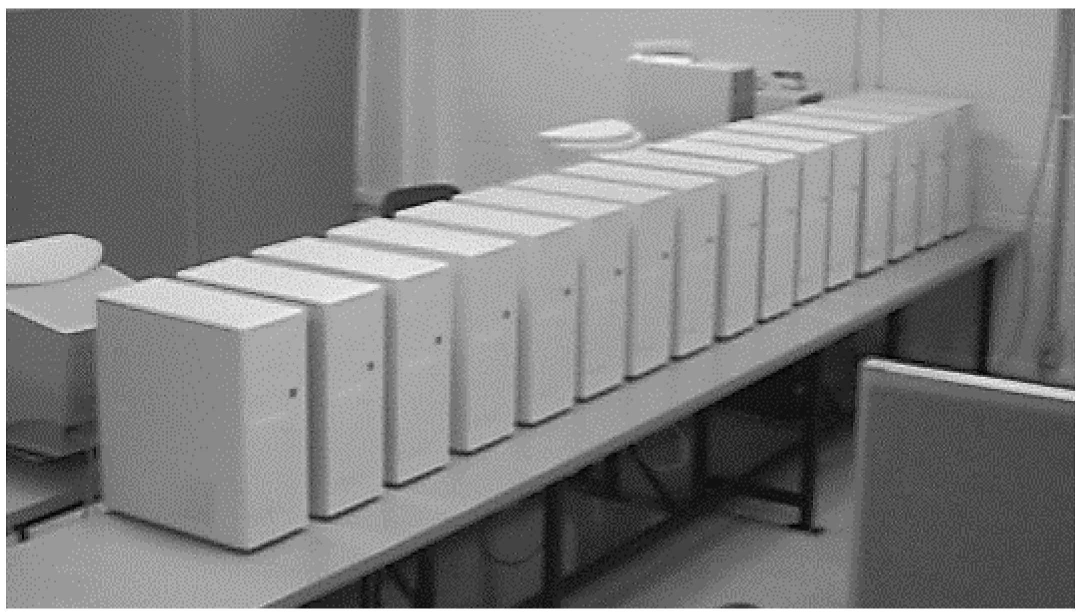


# Parallel Computer

- based on commodity hardware components and public domain software
- 16 dual Pentium II 400 MHz based PC's
- 11 dual Pentium II 500 MHz based PC's
- 100 Megabits/second switched ethernet

# Parallel Computer cont.

- total of 54 processors and 10.75 GB of main memory
- Compressible NSE solver achieved 1.55 Gflop/sec with a LU SSOR solver on a 100x100x100 structured grid on 32 processors
- GA optimization of a MHD diffuser completed in 30 hours.  Same problem would take 14 days on a single CPU



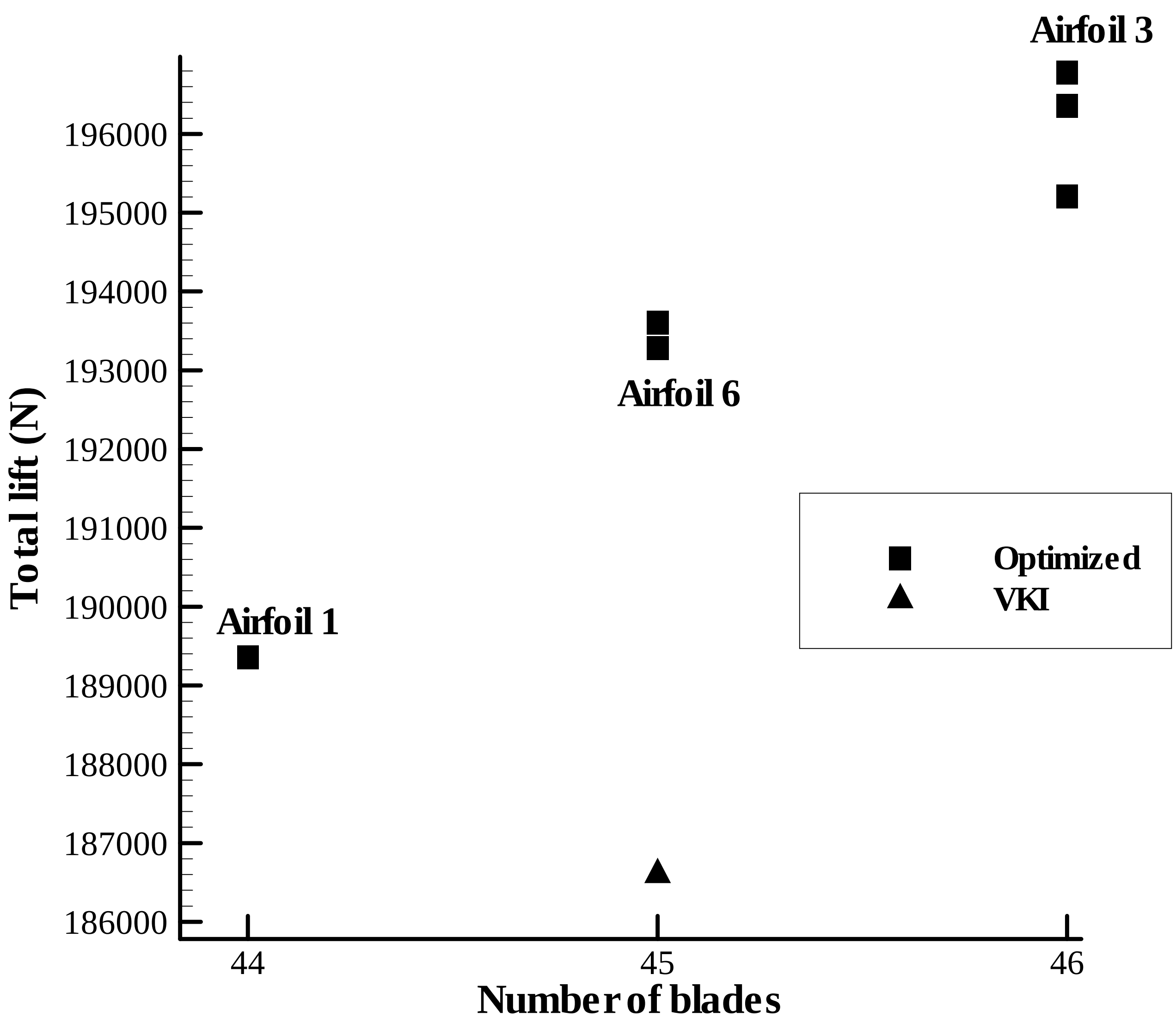
# Results

- Optimization program used 32 processors, 1 master and 31 slaves. MPICH library was used for synchronous communications between processors
- By 1000 calls, the front contained 1 feasible point

  By 2000 calls, the front contained 3 feasible points

  By 3000 calls, the front contained 7 feasible points
- Total computation time was about 50 hours
- A total of 5611 calls to the analysis code were made

# Computed Results: Pareto Front



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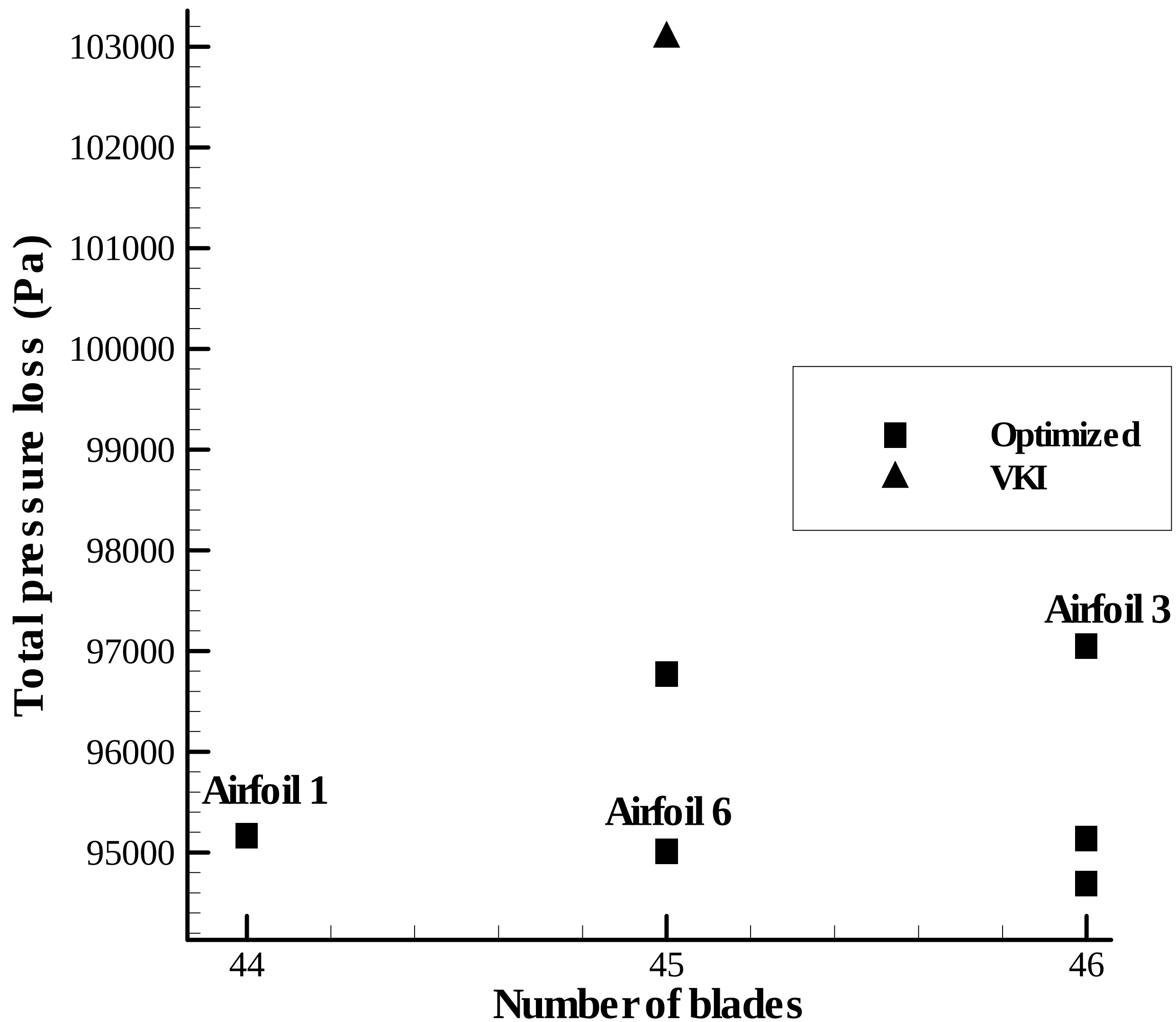

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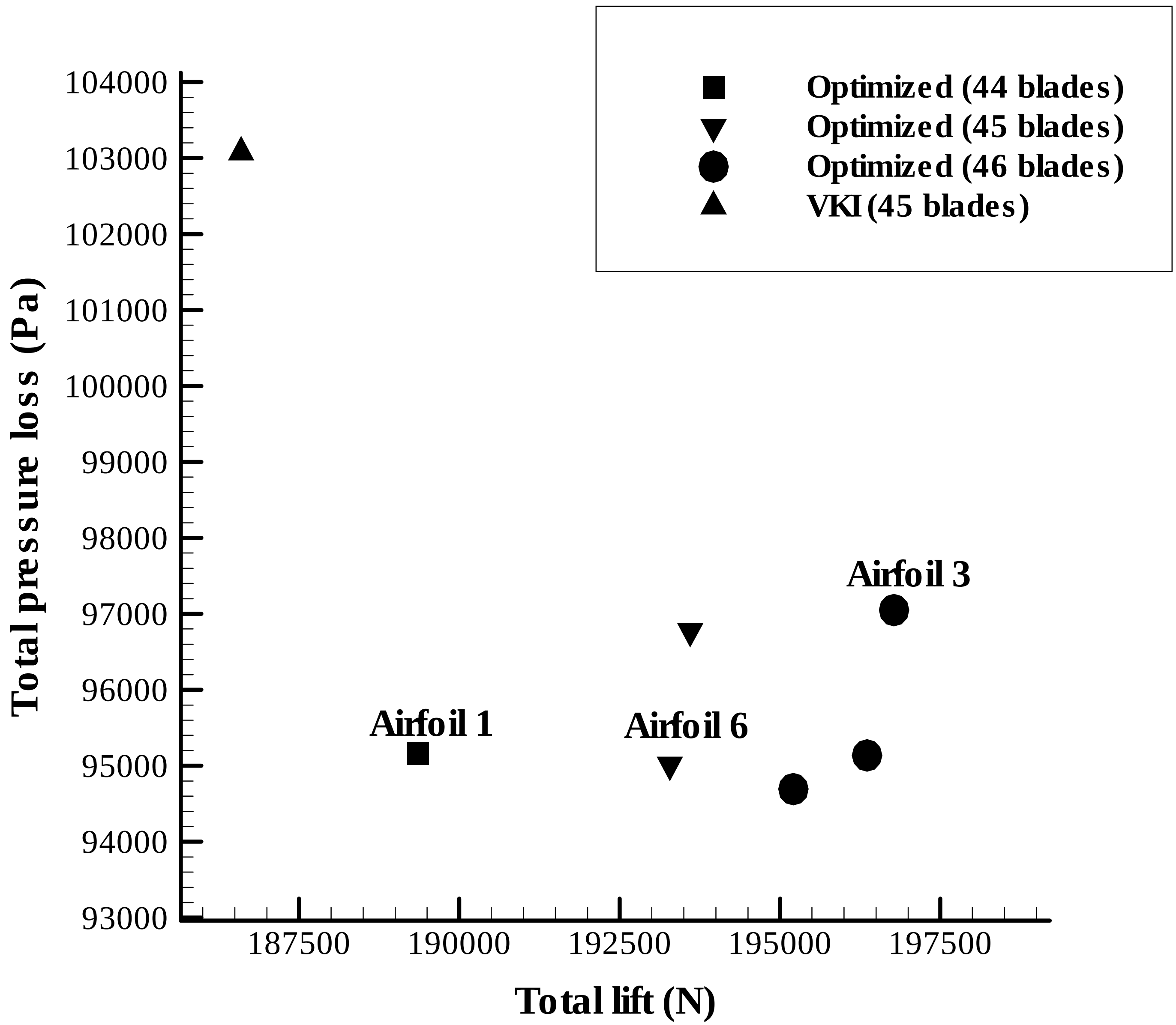
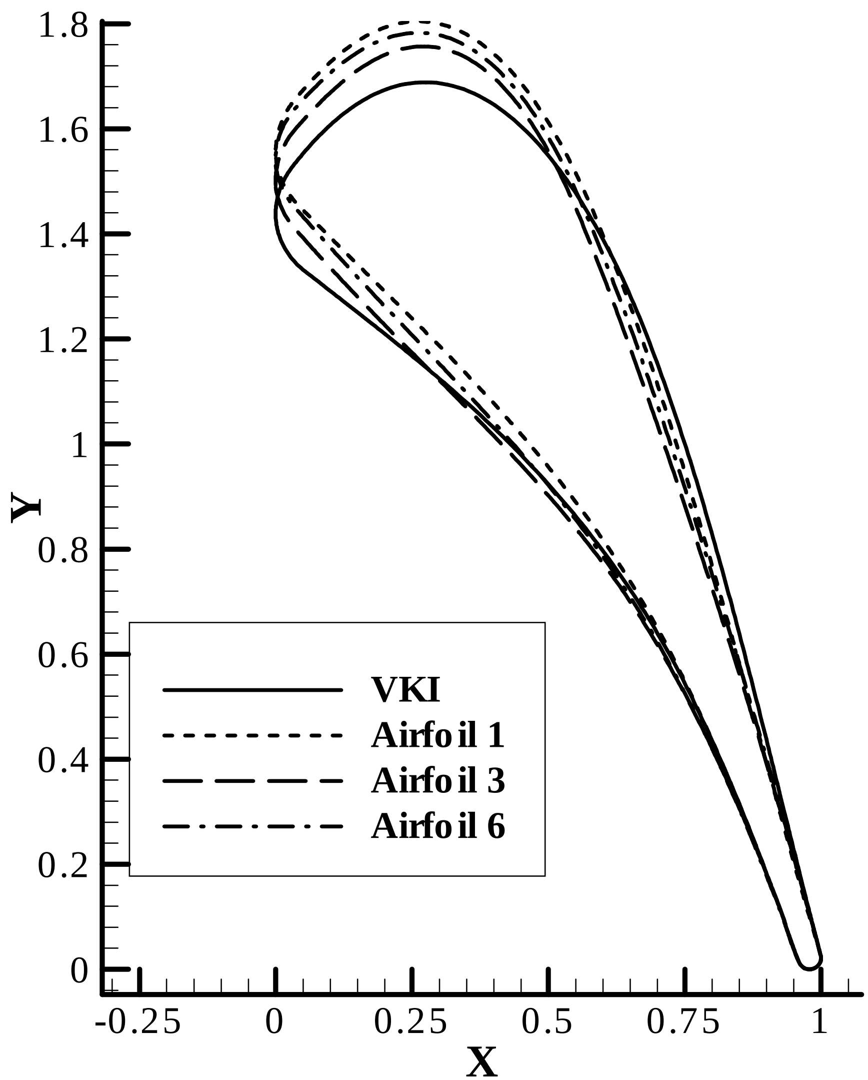


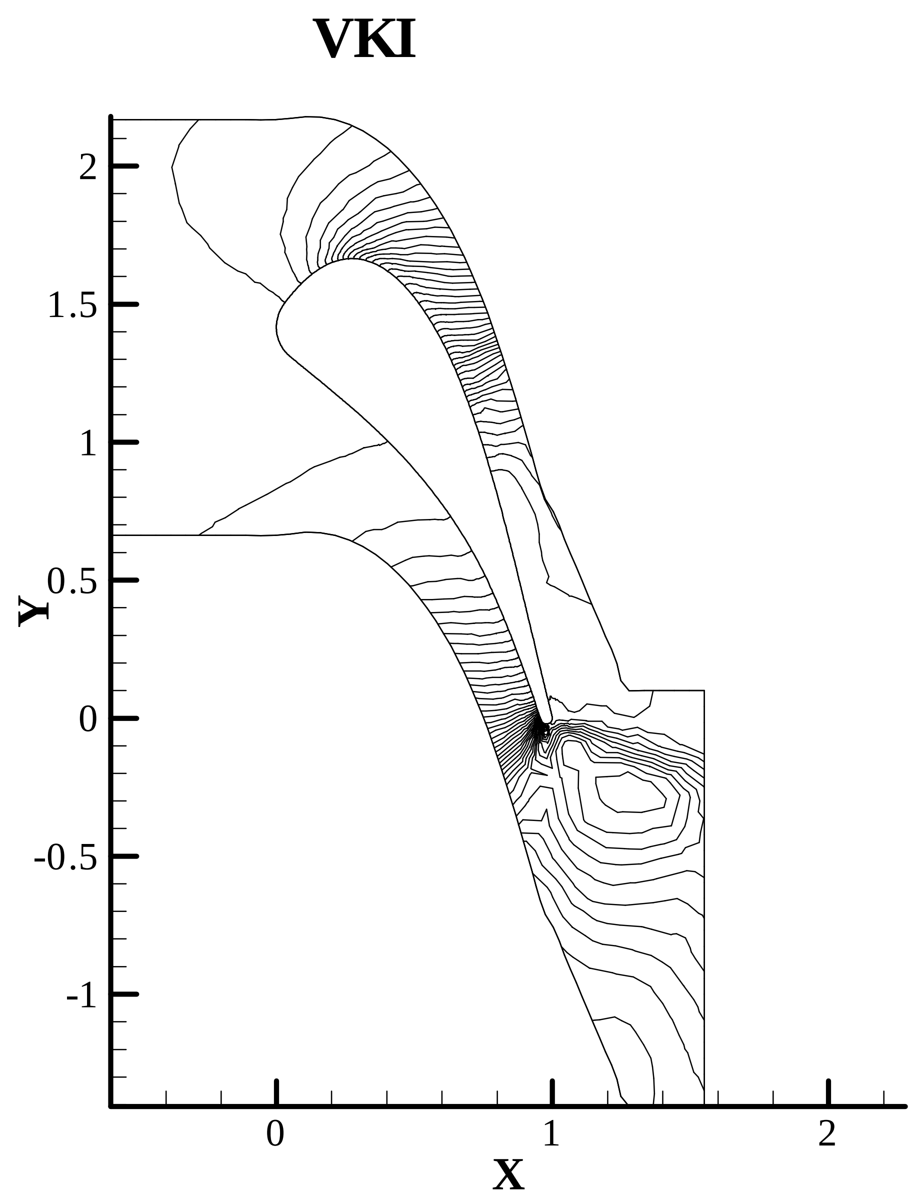
# Computed Results

| | VKI cascade | Cascade No.1 | Cascade No.3 | Cascade No.6 |
|---|---|---|---|---|
| Total pressure loss, Pa | 103078 | 95164 | 97050 | 95012 |
| Number of airfoils | 45 | 44 | 46 | 45 |
| Total loading, N | 186599 | 189359 | 196778 | 193228 |

# Computed Results

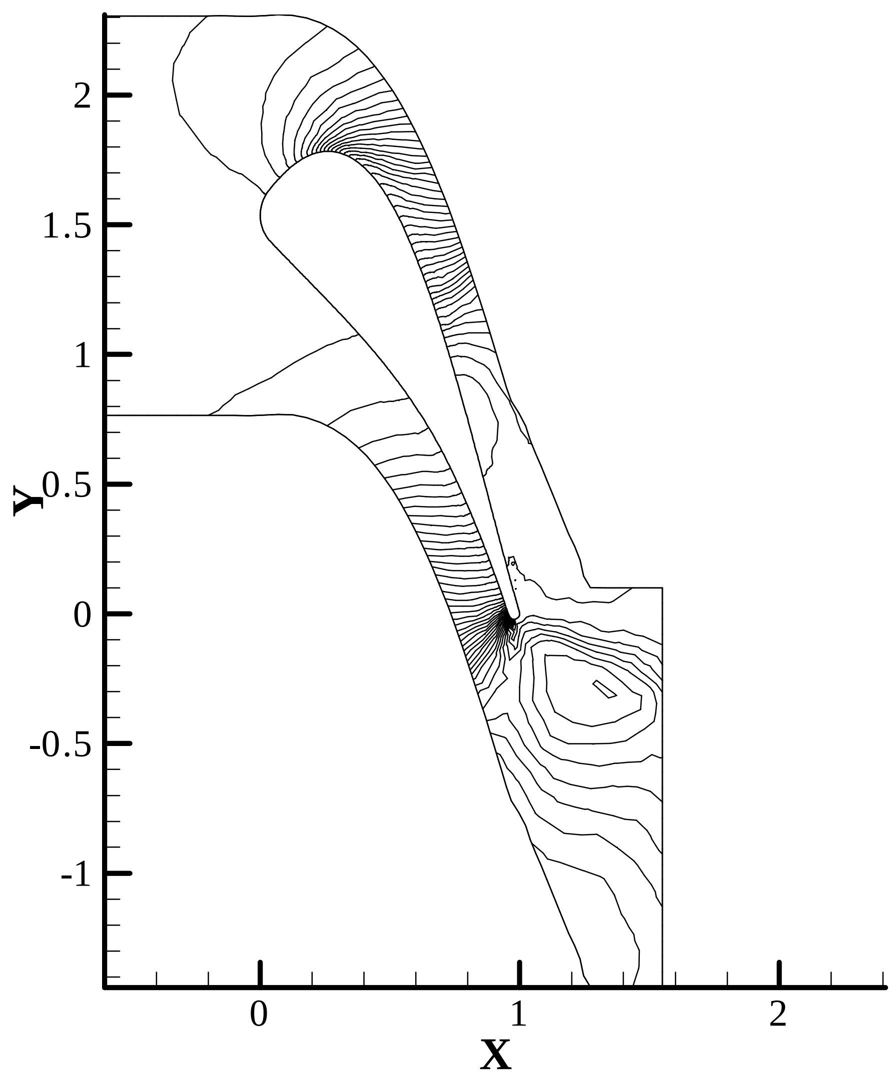


# Computed Results: Static Pressure

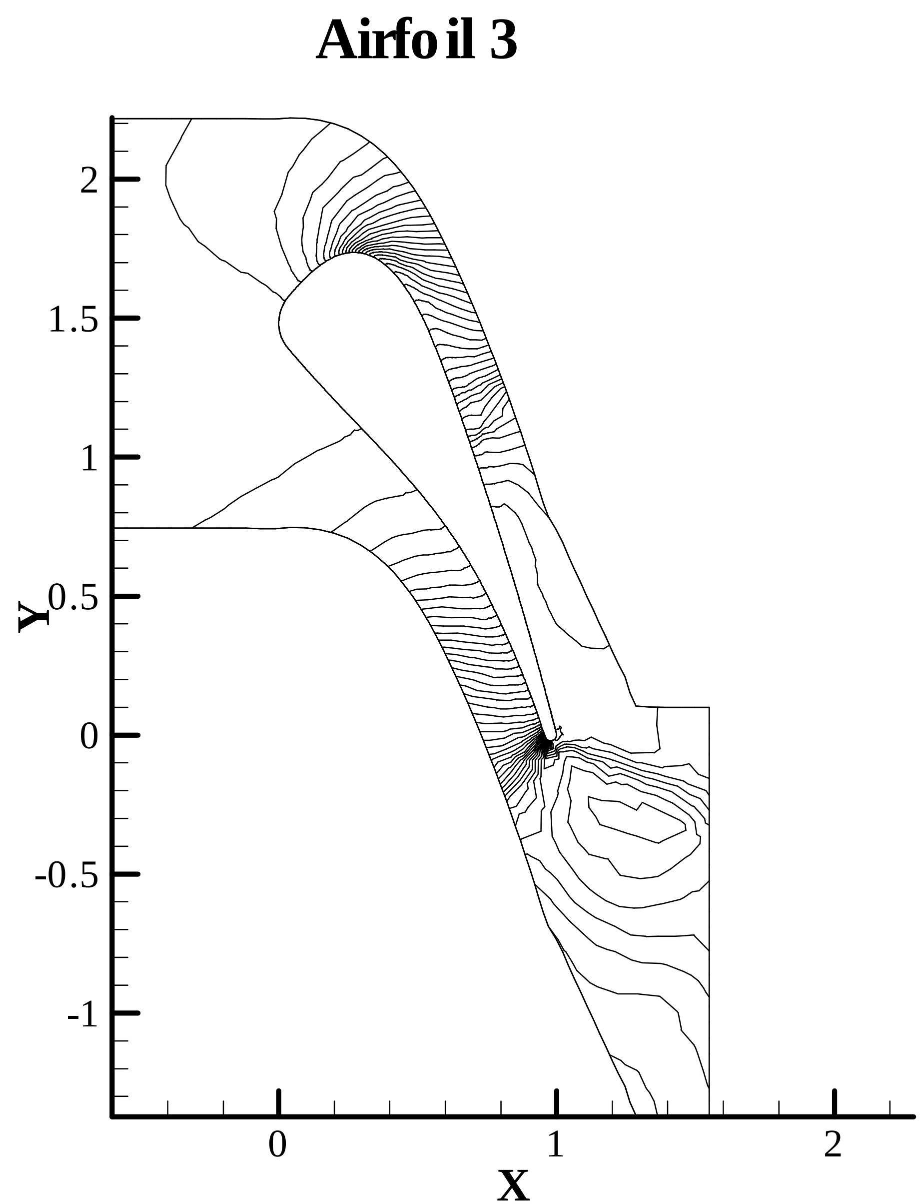


# Computed Results: Static Pressure

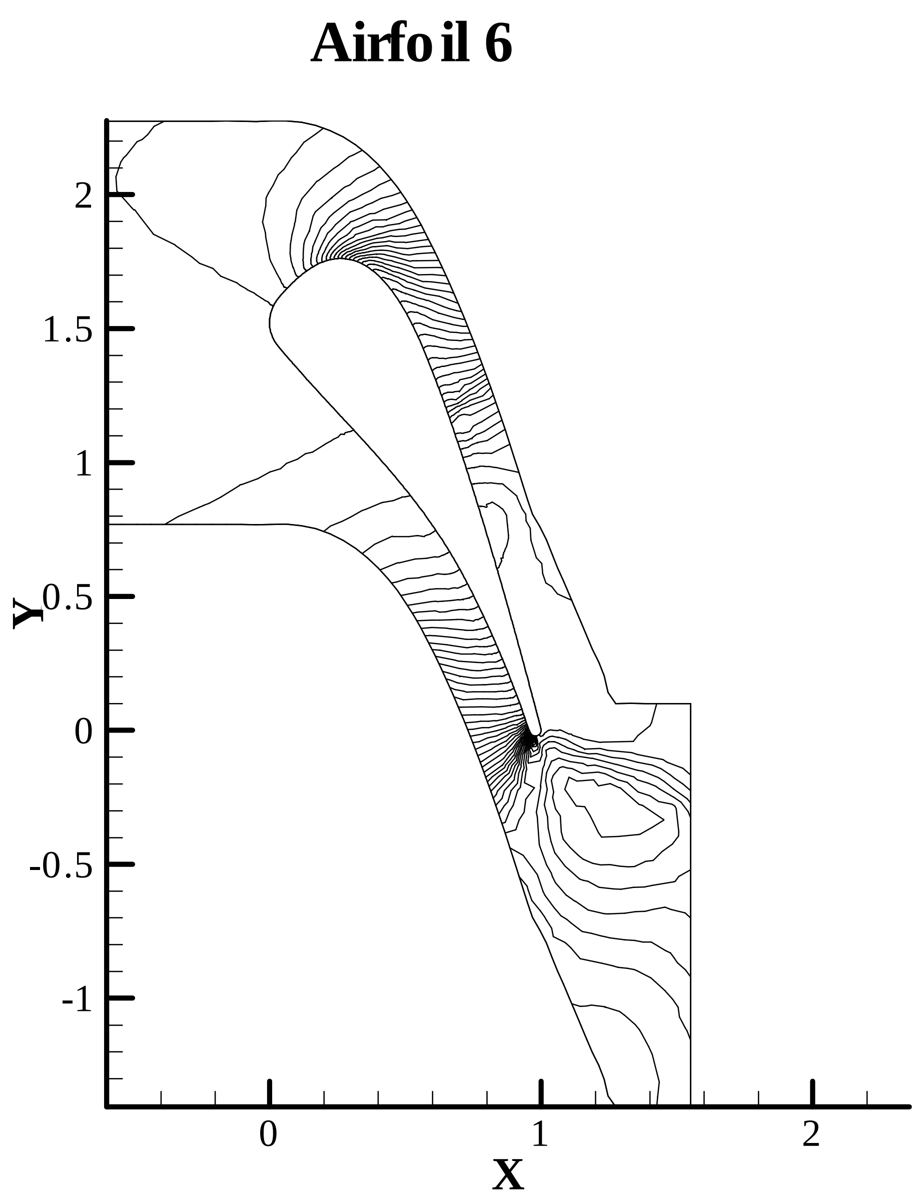
**Airfoil 1**



# Computed Results: Static Pressure



# Computed Results: Static Pressure



# Conclusions

It was demonstrated that it is possible to automatically design airfoil cascade configurations that will be more efficient and have fewer airfoils than the existing good cascades(VKI).

This design methodology can be extended to 2-D rotor cascades and to fully 3-D blade row configurations.