DETERMINATION OF UNSTEADY CONTAINER TEMPERATURES DURING FREEZING OF THREE-DIMENSIONAL ORGANS WITH CONSTRAINED THERMAL STRESSES

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Objectives

- Use finite element method(FEM) model of transient heat conduction and thermal stress analysis together with a Genetic Algorithm(GA) to determine the time varying temperature distribution that will cool the organ at the maximum cooling rate allowed without exceeding allowed stresses

Mathematical Model

Thermoelasticity

$$(\lambda + G)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\ \partial y} + \frac{\partial^2 w}{\partial x\ \partial z}\right) + G\nabla^2 u + X = 0$$

$$(\lambda + G)\left(\frac{\partial^2 u}{\partial x\ \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y\ \partial z}\right) + G\nabla^2 v + Y = 0$$

$$(\lambda + G)\left(\frac{\partial^2 u}{\partial x\ \partial z} + \frac{\partial^2 v}{\partial y\ \partial z} + \frac{\partial^2 w}{\partial z^2}\right) + G\nabla^2 w + Z = 0$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \qquad G = \frac{E}{2(1+\nu)}$$

$$X = -(3\lambda + 2G)\frac{\partial(\alpha\Delta T)}{\partial x} \qquad Y = -(3\lambda + 2G)\frac{\partial(\alpha\Delta T)}{\partial y} \qquad Z = -(3\lambda + 2G)\frac{\partial(\alpha\Delta T)}{\partial z}$$

Mathematical Model

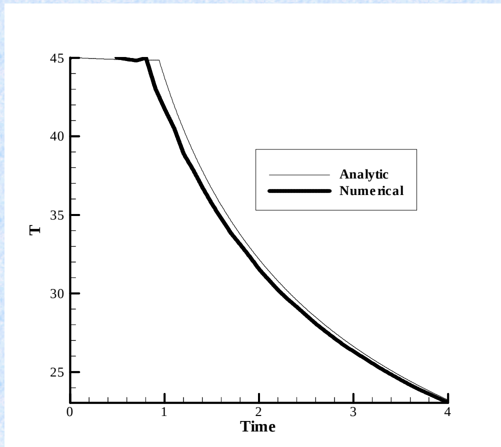
Heat Conduction

$$\frac{\partial(\rho C_{effective} T)}{\partial t} = \nabla \cdot (k\nabla T)$$

$$\rho C_{effective} \approx \frac{dH}{dT}$$

$$\frac{dH}{dT} = \left[\frac{\left(\frac{dH}{dx}\right)^2 + \left(\frac{dH}{dy}\right)^2 + \left(\frac{dH}{dz}\right)^2}{\left(\frac{dT}{dx}\right)^2 + \left(\frac{dT}{dy}\right)^2 + \left(\frac{dT}{dz}\right)^2}\right]^{\frac{1}{2}}$$

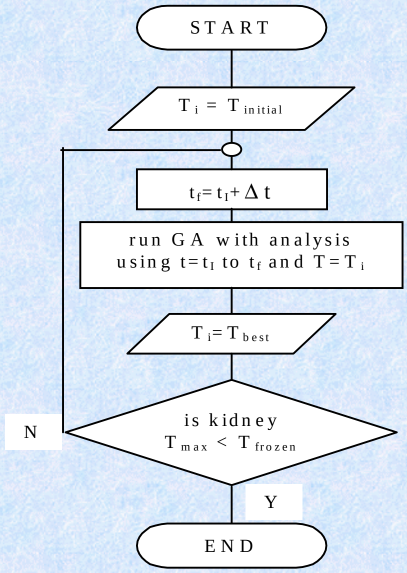
Comparison with Analytic Solution



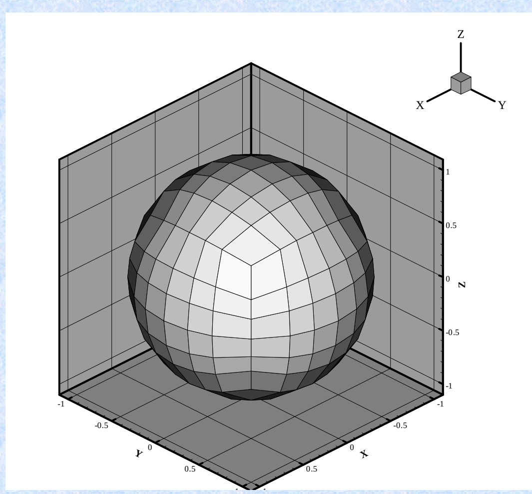
Geometry and Material Properties

- Actual Geometry from MRI or other sources was not available so a kidney like object was constructed from analytical functions
- Some material properties are available in various publications but some had to be estimated for this test case
- The procedure can be easily repeated when accurate geometry and material data are available

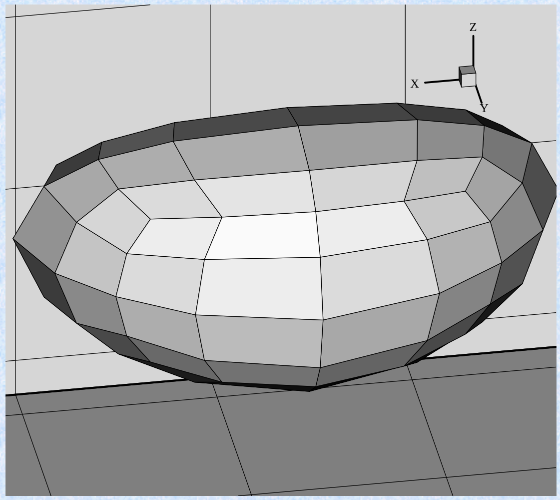
Algorithm



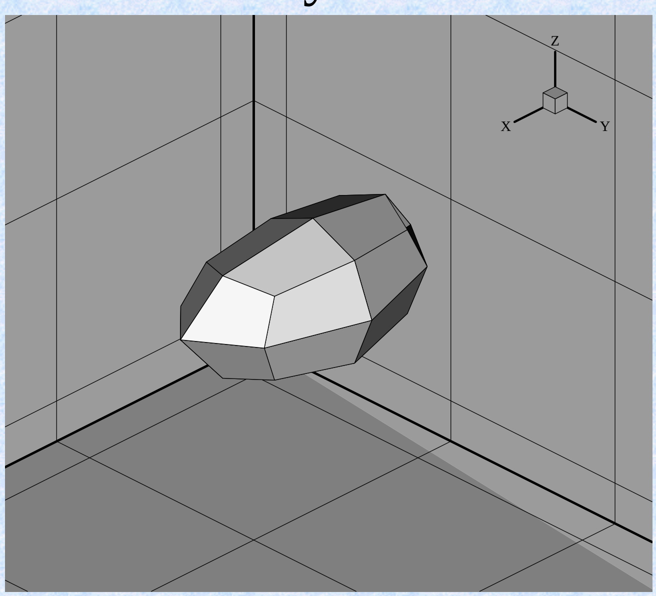
Outer Boundary



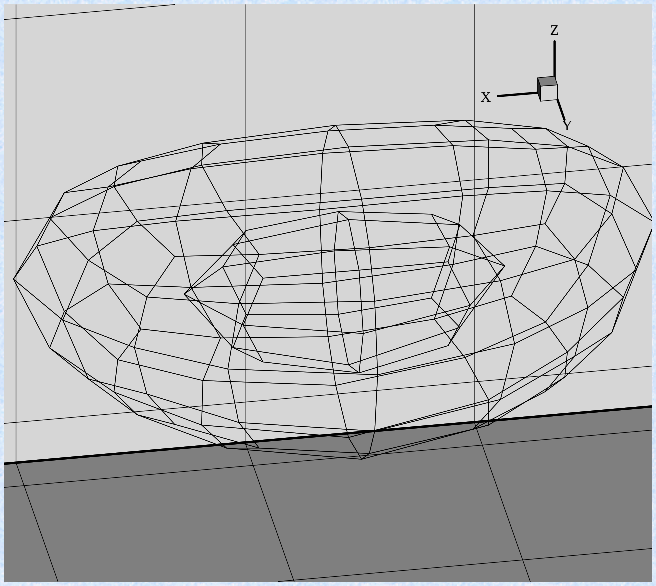
Kidney Medula



Kidney Cortex



Kidney Cortex and Medula



Parameterization

- Outer sphere divided into six equally spaced patches.
- Each patch is covered by biquadratic Lagrange polynomials with nine nodes
- 26 design variables total
- Allowed to vary from 20 to -30 C

Objective Function

Minimize average heating rate with max. stress as a constraint

$$F = \frac{\Delta T}{\Delta t} + P\left(\frac{\sigma_{max}}{\sigma_{yield}}\right)^2$$

If(max stress > yield stress)

P = 100

else

P = 0

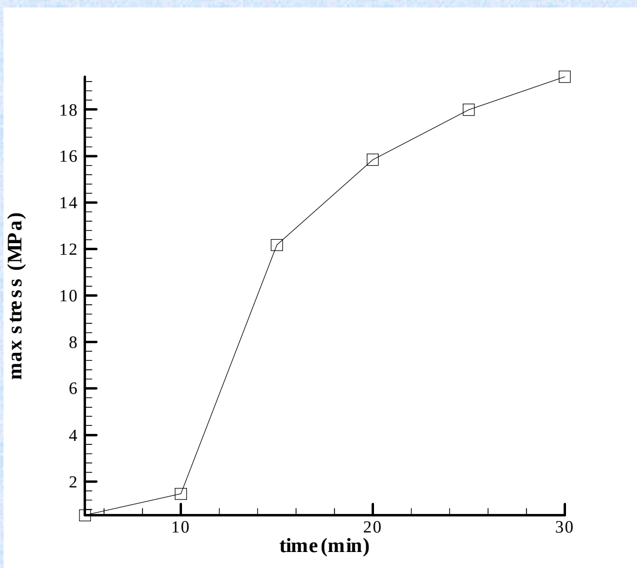
Numerical Results

- Time interval for outer loop was 5 minutes
- Time step for analysis was 15 seconds
- Was run for 30 minutes(6 surface temperature distribution optimizations)
- 5184 parabolic tetrahedral element grid was used for heat conduction
- 5184 linear element grid was used for stress analysis

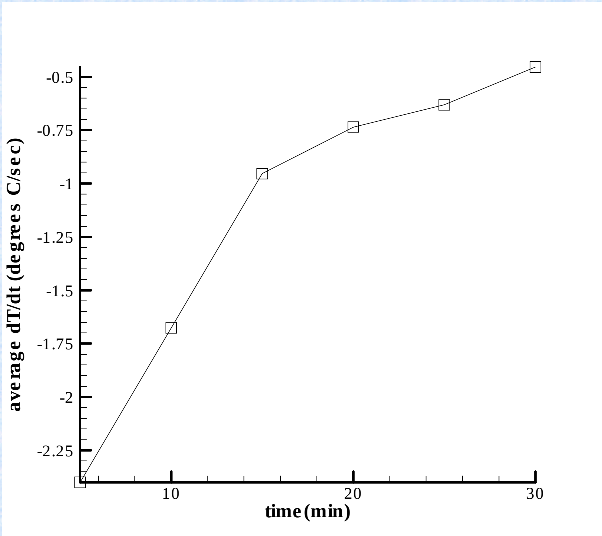
GA Parameters Used

- Each design variable represented with 4-bit binary string
- Uniform crossover operator was used
- 50% probability of crossover, 2% probability of mutation
- 15 generations were run for each optimization
- Used a population size of 31
- Was run on a 32 processor parallel computer composed of commodity components
- One optimization takes ~45 minutes

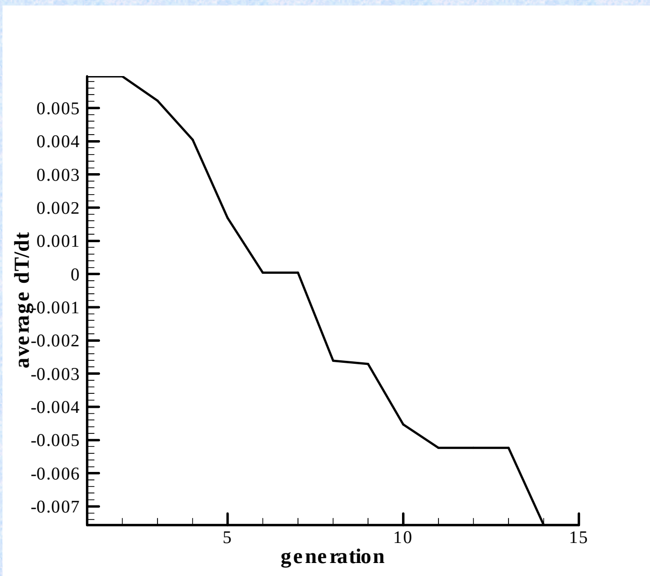
Numerical Results



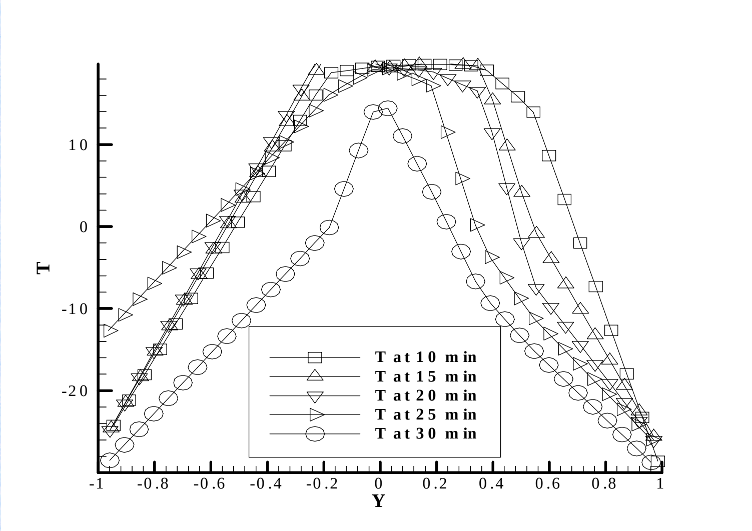
Numerical Results



Numerical Results



Numerical Results



Conclusion

- An algorithm was developed for the inverse determination of surface temperature distributions in the freezing of a kidney organ
- Current results show that after 30 minutes the kidney is not entirely frozen and more time intervals are needed
- This simulation process was made practical through the use of parallel optimization algorithms(GA) and cheap parallel computer composed of PC components